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GRAVITY AND SPATIAL INTERACTION MODELS

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1. GRAVITY MODEL: OVERVIEW

Spatial interaction is a broad term encompassing any movement over space that results from a human process. It includes journey-to-work, migration, information and commodity flows, student enrollments and conference attendance, the utilization of public and private facilities, and even the transmission of knowledge. Gravity models are the most widely used types of interaction models. They are mathematical formulations that are used to analyze and forecast spatial interaction patterns.

The gravity model as a concept is of fundamental importance to modern scientific geography because it makes explicit and operational the idea of relative as opposed to absolute location. All things on the face of the earth

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can be located in absolute terms by longitude and latitude coordinates, and the absolute position of things can be related to each other by reference to such coordinates. Distances can be specified in these absolute terms. It is then possible to talk about one location as being "five miles from New York City" and another as being "five miles from Bloomington, Indiana." In absolute terms, these two locations are equal in that they are both five miles from an urban area. In relative terms, however, these locations are significantly different in a multitude of ways (for example in terms of access to shopping, access to job opportunities, access to museums and theaters, access to rural life-styles, or access to wilderness opportunities). Each of these significantly differentiates absolute location from relative location. The gravity model allows us to measure explicitly such relative location concepts by integrating measures of relative distance with measures of relative scale or size.

The importance of the relative location concept and spatial interaction can be seen in the application and refinement of the gravity model over the past fifty years. Its continued use by city planners, transportation analysts, retail location firms, shopping center investors, land developers, and urban social theorists is without precedent. It is one of the earliest models to be applied in the social sciences and continues to be used and extended today. The reasons for these strong and continuing interests are easy to understand and stem from both theoretical and practical considerations.

Social scientists are interested in discovering fundamental and generalizable concepts that are basic to social relationships. One of the distinguishing features of human behavior is the ability to travel or move across the face of the earth and to exchange information and goods over distance. Such exchange processes are referred to generically as interaction, and that which occurs over a distance occurs over space. Hence, the general term "spatial interaction" has been developed to characterize this common type of geographic behavior. Shopping, migrating, commuting, distributing, collecting, vacationing, and communicating usually occur over some distance, and therefore are considered special forms of this common social behavior—spatial interaction. We seek here to describe fundamental characteristics that underlie all these forms of social behavior. We will make generalizations about those characteristics that explain or predict similar geographic behavior. Our goal is to demonstrate that spatial interaction models can be considered as the basis of important and useful social theories. The gravity model is one example of a spatial interaction model.

The gravity model, which derives its name from an analogy to the gravitational interaction between planetary bodies, appears to capture and interrelate at least two basic elements: (1) scale impacts: for example, cities with large populations tend to generate and attract more activities than cities with small populations; and (2) distance impacts: for example, the farther places, people, or activities are apart, the less they interact.

These concepts are used by urban social analysts to explain why land values are high in the central areas of cities and at other easily accessible points (Hansen 1959) and why land values are higher in larger cities than in smaller cities. They are used to explain why some public service or retail locations attract more users or customers than do others and to explain the way in which shopping centers impact the areas about them in terms of traffic and customer flows. On a larger scale, they are used to explain the movement of population in the form of migrants, visitors, business and commercial travelers, and the movement of information in the form of mail, telecommunications, and data transfers. In practical terms, these are important topics for many kinds of decision makers, both public and private; a model that purports to reduce the risk in making large capital decisions related to these topics obviously is valuable.

The applications to which the gravity model has been put are not limited to transportation, marketing, retailing, and urban analysis. In archaeology Hallam, Warren, and Renfrew (1976) have used it to help identify probable prehistoric exchange routes in the western Mediterranean, while Jochin (1976) has used it to examine the location and distribution of settlement among hunting-and-gathering peoples. Tobler and Wineberg (1971) used related methods to develop suggestions about the location of lost cities. More recently Clark (1979) has used a type of gravity model analysis to explain archeological data on the flow of goods. In a related context, Kasakoff and Adams (1977) have used location and anthropological information together with a gravity model formulation to explain marriage patterns and clan ties among Tikopians. Trudgill (1975) made a strong argument for the wider use of this method in linguistics, while Hodder (1980) has made a general plea for the wide use of this technique in trying to understand patterns and potential patterns in the spatial organization of both historic and prehistoric activities.

The Model

Figure 1.1 illustrates the basic relationships inherent in gravity models. Compare the expected level of flows between city x and city y with that between cities x and z. Without further information our intuitive expectation would be that the flows between x and y would be larger; y and z are the same distance from x—800 miles—but y has a population of 2 million while z has a population of only 1 million. If interaction were a function of pairs of individuals in any two cities then the potential sets of pairs between x and y is larger than between x and z (2,000,000 × 2,000,000 vs. 2,000,000 × 1,000,000). The potential pairs of interactions would be twice as great in the case of x and y than in the case of x and z. This is the *multiplicative* impact of scale on interaction.

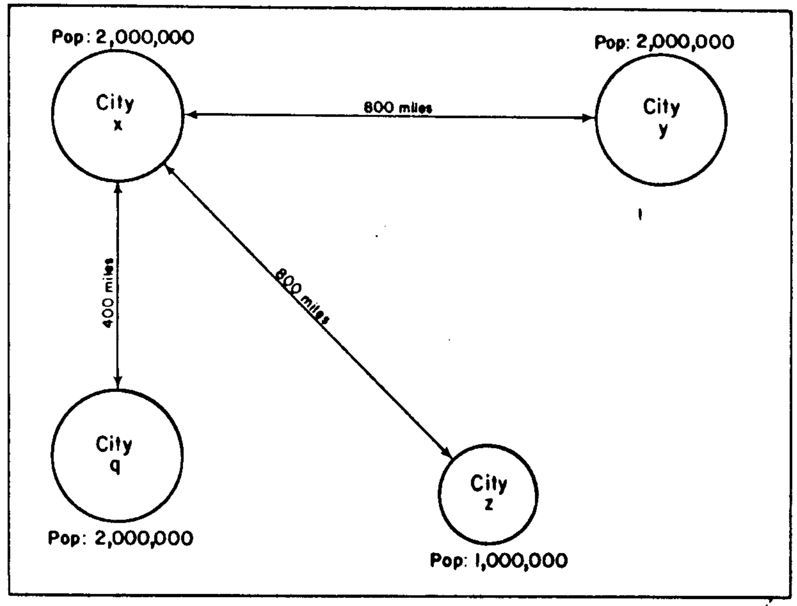
The impact of distance can be demonstrated by comparing the expected levels of flows between x and y, and x and q. The sizes of y and q are the same, so scale is constant. However, without further information we would expect more flows between x and q than between x and y because we would expect the flows between any two points to decline as distance increases. If this decline is proportional to distance, then with scale held constant we would expect half as much interaction between x and y as between x and q.

To generalize, let the scale of each city, population, be represented by P, and the distance between cities be represented by d. Each pair of cities is designated by the subscripts i and j. Interaction between any pair of cities is specified as T_{ij}. The interaction can be expressed as a ratio of the multiplied populations over the distance between any pair of cities,

$$T_{ij} = P_i P_j / d_{ij} \qquad [1.1]$$

Modifications

Three fundamental modifications need to be made to the basic model in equation 1.1. First, the distance element is adjusted by an exponent to indicate whether the impact of distance is proportional or not. For example, the cost per mile of traveling may decrease with distance, as in air travel. Obviously the operational effect of distance would therefore not be directly proportional to airline miles and the negative aspect of distance would need to be reduced or dampened so that the model properly reflects its effects. On the other hand the effect of distance may be underestimated by mileage because the opportunity to know people in cities far away may be reduced by language, culture, and information. The impact of distance may be greater than that indicated by use of straight line mileage in the model. This "distance decay" or "friction of distance" effect will vary depending on the flows being examined—air transportation as opposed to private automobile transportation, for example. Even though distance will always have a negative influence on interaction, in some cases it may be



NOTE: This illustration demonstrates the basic principles and trade-offs between the effects of scale (population) and distance (mileage) upon expected interactions between places.

Figure 1.1 The Gravity Model Principles

more negative than in others. An exponent on the distance variable, d_{ij}^{β}, allows us to represent this variability.[1] A large theoretical and empirical literature has developed around the definition of the "correct" exponent.

Much of the literature that focused on deriving the correct exponent for the gravity model formulation was stimulated by physical science interpretations, including the Newtonian analogy where the square of distance, d_{ij}^2, is the appropriate power function. In empirical analysis, however, the exponent is generally interpreted as the responsiveness of interaction to spatial separation and is expected to vary in terms of social context. Larger exponents indicate that the friction of distance becomes increasingly important in reducing the expected level of interaction between centers. Figure 1.2, part a, indicates the impact of small and large values of the exponent β on the distance variable. Other things being equal we would expect distance

CALCULATION OF MARKET BOUNDARY BETWEEN TWO CITIES

Application of a gravity Model (Reilly's Law of Retail Gravitation) to calculating the market boundary between two cities.

Input: (Enter information required below)

Population of City 1 =	4,000
Population of City 2 =	16,000
Distance between cities =	12

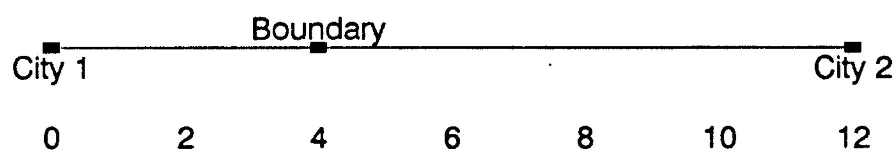
Results:

Distance from City 1 to Market Boundary =	4.00
Distance from City 2 to Market Boundary =	8.00

-- Press F10 to See Graph --

Market Boundary Between Cities 1 and 2

Market Boundary According to Gravity Model



Note: Do Not Try to Do this Problem Until I have gone over it in Class & Assigned it (Unless you want to)

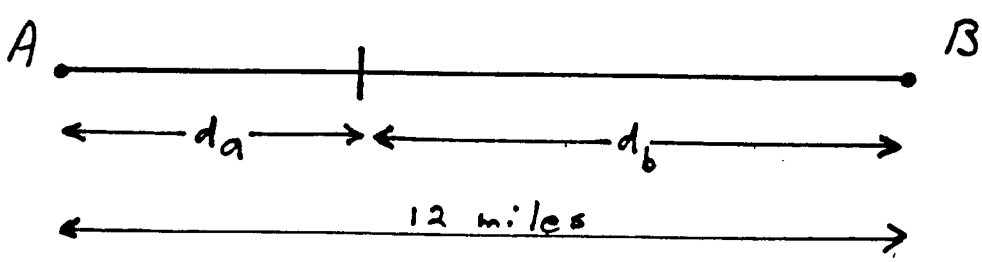
Stipak

In class we discussed Reilly's law of retail gravitation:

$$F = K \frac{P}{d^2}$$

where: F = retail attractiveness
P = size of town
d = distance to town

Using this law, determine the point on a straight line between two towns 12 miles apart which is the market boundary between the two towns. Town A has a population of 4000, Town B a population of 16,000.



Note: $$d_a = \frac{d\sqrt{P_a}}{\sqrt{P_b} + \sqrt{P_a}}$$